# Appendix A - Grid Study

A grid independent study was performed to determine an adequate mesh size for validation of the numerical results with the lowest computational cost. As described in Section 4, the prime numerical output from the model was the volume fraction at different locations in the ground domain between the pipeline hole and the atmosphere interphase. Hence, the grid independent analysis was performed by refining the mesh until the variations in the volume fraction at five points in the ground domain were negligible (Fig. A.1): in the vicinity of the hole (near hole location), near the atmosphere (surface point), a point situated in the ground between the hole and atmosphere (subsurface location), to the left and right corners of the soil (left subsurface location and right subsurface location, respectively). The mass flow rate was evaluated at the ground level for additional justification of the selected mesh.

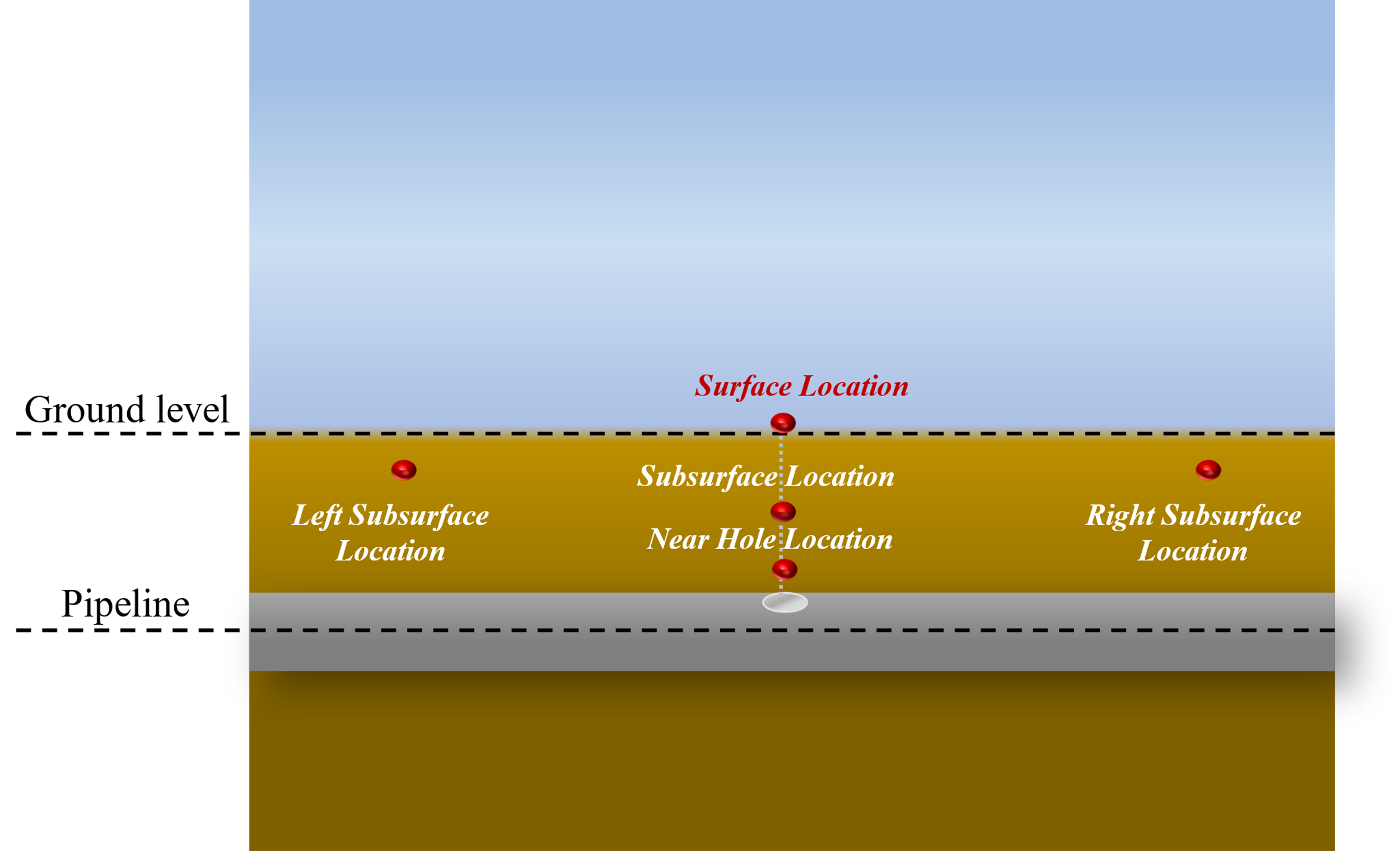
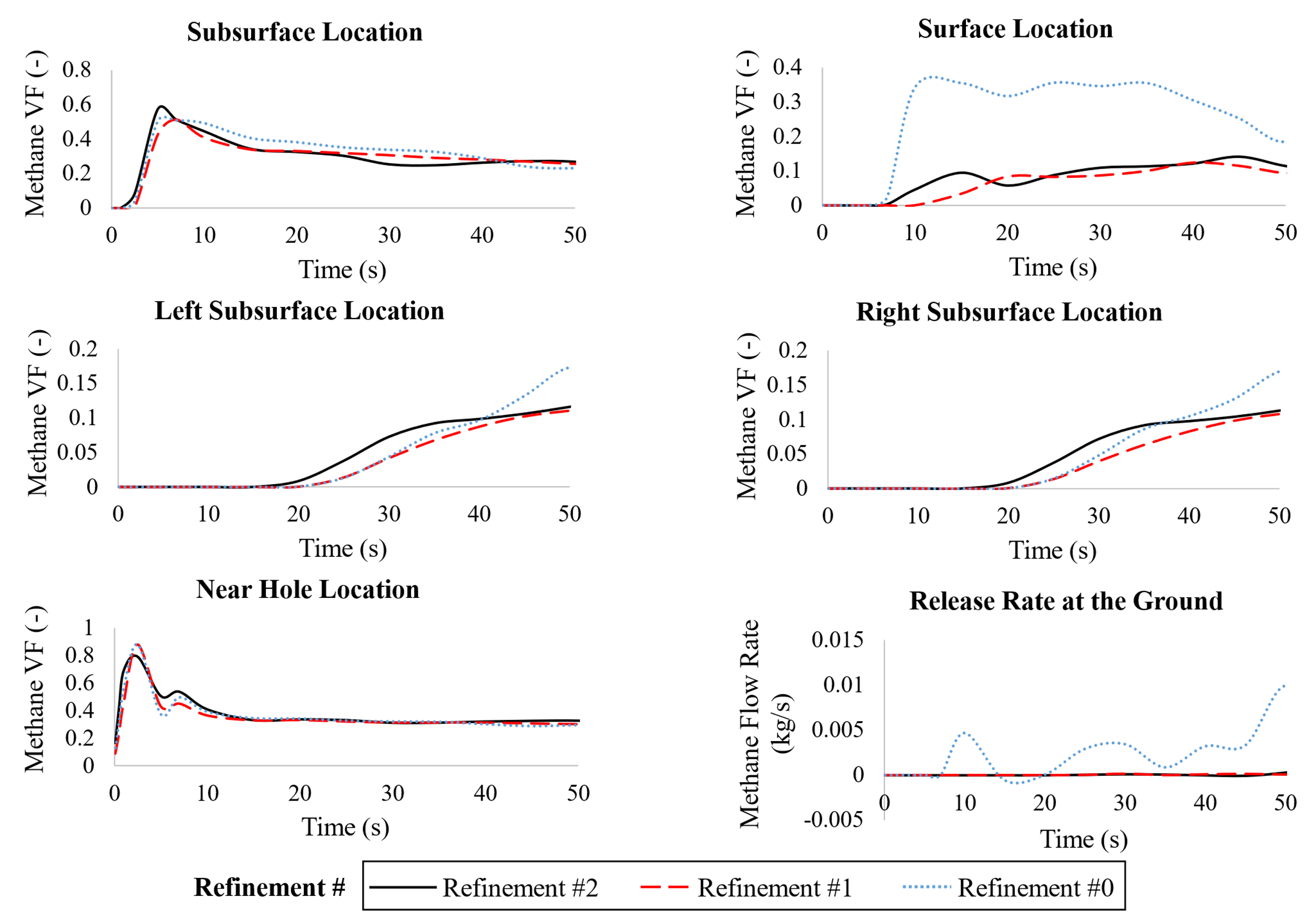


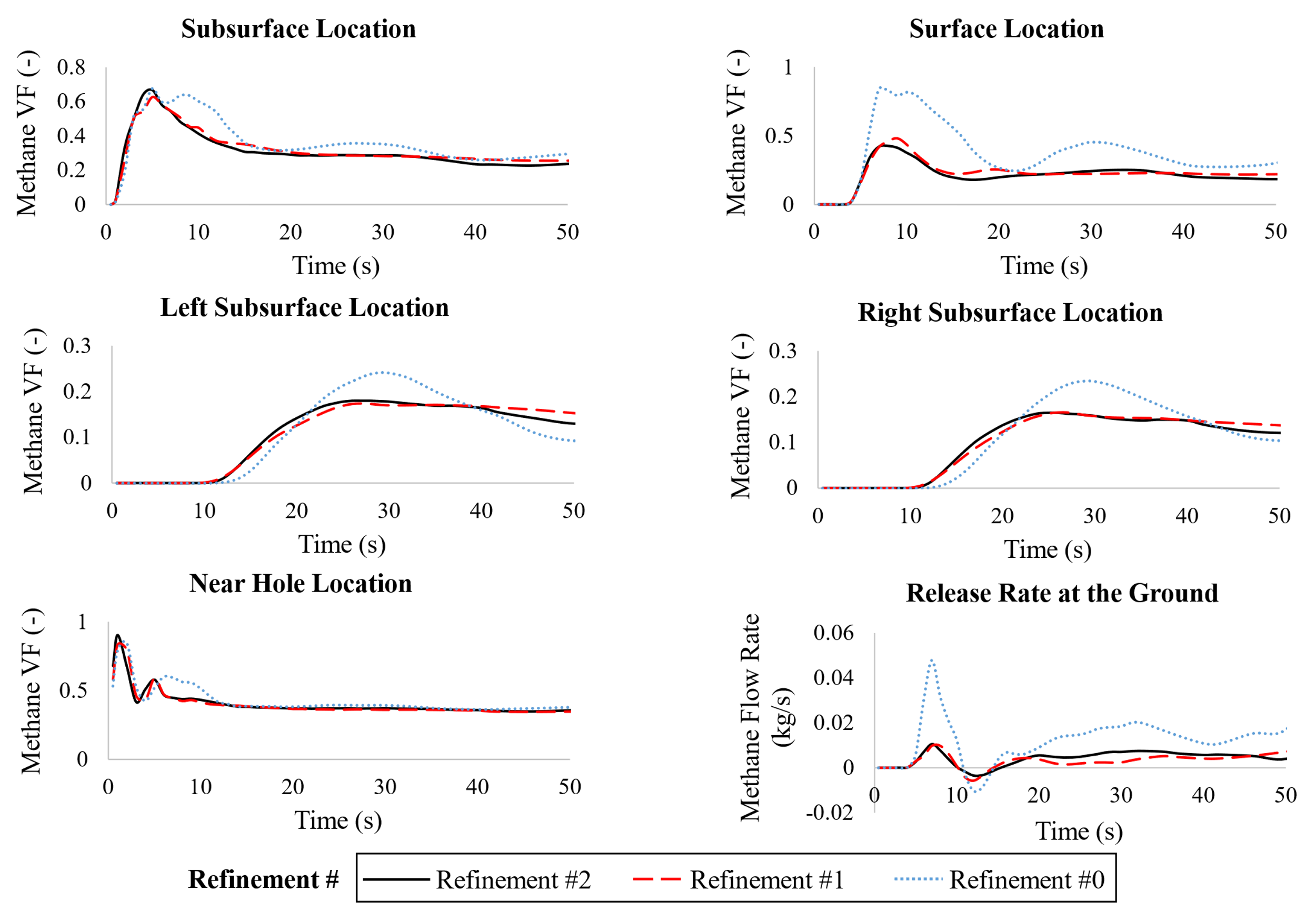
Fig A.1. Monitoring locations to evaluate the volume fraction

Three meshes are reported in the mesh analysis from the coarsest to the finest entitled refinement #0 (870,942 elements), refinement #1 (1,402,326 elements) and refinement #2 (2,365,353 elements). The meshes were tested at two pressures of 15 bar (Fig. A.2a) and 40 bar (Fig. A.2b).

As shown in Fig. A.2, while refinements #1 and #2 agree both in trend and values at all locations, the coarsest mesh yields some discrepancies in a few locations. The average error between the results from refinement #1 and refinement #2 was acceptable and within the experimental error margin. In particular, the average error for the 15 bar was around 11% (3% - 7% for the locations above the hole, and ~20% for the left and right edges of the domain), and around 2% for the 40 bar (with no significant variations among the different locations). Therefore, the mesh of refinement #2 was considered for determining the results reported herein, as the more accurate and computationally affordable with the available resources. Note that simulations with a fourth refinement of ~3.950 million elements was tested but required a simulation time of more than a few weeks, hence was discarded from the analysis.



(a)



(b)

Fig. A.2. Time-series of the release rate at the ground and the methane VF at the subsurface, surface, left subsurface, right subsurface and near hole locations for the different mesh refinement at (a) 15 bar and (b) 40 bar.

# Appendix B – Mathematical multiphase model

The continuity equation for each phase *q* (*q = gas g, solid s*) is:

(B.1)

The momentum balance for the gas phase *g* is:

(B.2)

Similarly, the momentum balance for the solid phase *s* is:

(B.3)

The gas and solid phases stress tensors are represented by the following, respectively:

(B.4)

(B.5)

Thebulk viscosity accounts for the resistance of the granular particles to compression and expansion, and it was described by [Lun *et al.* (1984)](#_ENREF_2):

(B.6)

The shear viscosity is comprised of the granular (or kinetic) viscosity, collisional viscosity, and frictional viscosity.

(B.7)

The granular viscosity accounts for the kinetic fluctuations and is frequently described by the Gidaspow ([Gidaspow *et al.*, 1992](#_ENREF_1)) and Syamlal-O’Brien ([Syamlal *et al.*, 1993](#_ENREF_5)) models, respectively:

(B.8)

(B.9)

The collisional viscosity accounts for the collisions of the particles, and was described by the expression of [Gidaspow *et al.* (1992)](#_ENREF_1):

(B.10)

The frictional viscosity accounts for the viscous-plastic transition at maximum solid volume fraction, and was modelled with the expression of [Schaeffer (1987)](#_ENREF_3):

(B.11)

The solids pressure is composed of a kinetic term and another term due to particles collisions, and is represented by [Lun *et al.* (1984)](#_ENREF_2)

(B.12)

The radial distribution function is a correction term that modifies the probability of collision when close to reaching the packing limit and is represented by [Lun *et al.* (1984)](#_ENREF_2):

(B.13)

The granular temperature conservation equation is:

(B.14)

: the generation of energy by the solid stress tensor

: the diffusion of the energy

: the rate of energy dissipation due to particles collisions

: the transfer rate of kinetic energy due to random collisions between the gas and solid phases

The simplified expression – algebraic form – neglects mainly the convection and diffusion terms to yield the following:

(B.15)

Where the collisional dissipation of energy is represented by [Lun *et al.* (1984)](#_ENREF_2):

(B.16)

The energy exchange between the gas and solid phases is given by [Gidaspow *et al.* (1992)](#_ENREF_1):

(B.17)

The interphase moment exchange coefficient available in the different conversation equations is described by [Schiller and Naumann (1935)](#_ENREF_4):

(B.18)

(B.19)

(B.20)

(B.21)

(B.22)

# References

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